PAG: a GAP package for constructing combinatorial objects with prescribed automorphism groups*

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University of Zagreb, Croatia

8.5.2026.

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Funded by the European Union
NextGenerationEU

Introduction

Source: https://www.addictioncenter.com

PAG

Prescribed Automorphism Groups

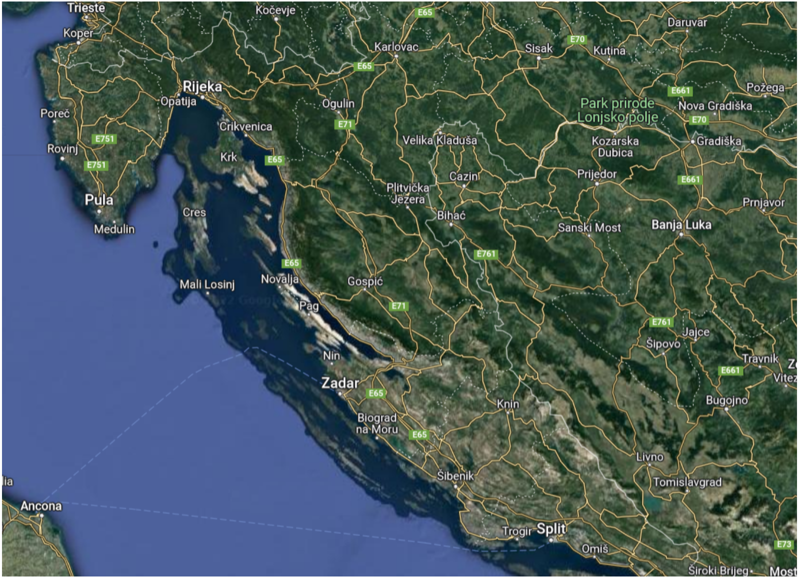
0.2.5

Abstract

PAG is a GAP package for constructing combinatorial objects with prescribed automorphism groups.

https://vkrcadinac.github.io/PAG/

The island Pag



Source: https://www.google.com/maps/place/Pag

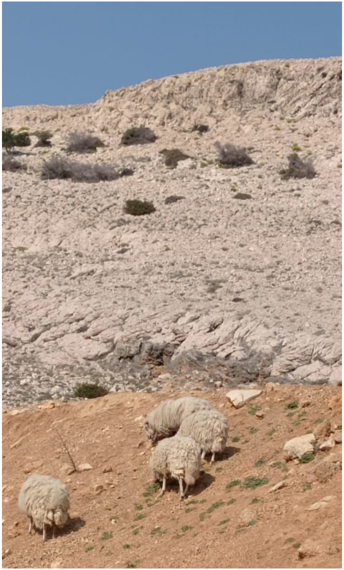
The island Pag

Source: https://www.google.com/maps/place/Pag

The island Pag – North shore

The island Pag – North shore

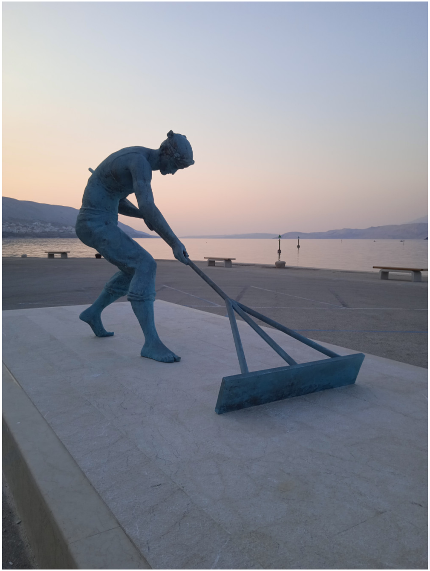
The island Pag – Sheep



The island Pag – Cheese

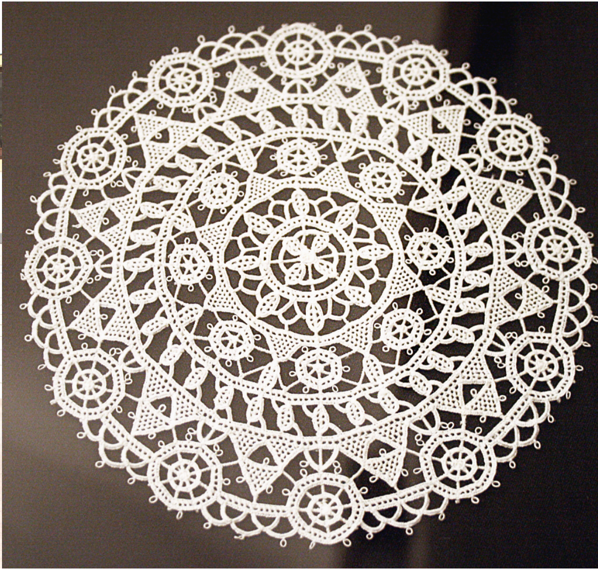
Source: https://www.paskasirana.com

The island Pag – Salt



Source: https://solana-pag.hr

The island Pag – Lacemaking



Source: https://en.wikipedia.org/wiki/Pag_(town)

The island Pag – Lacemaking

United Nations Educational, Scientific and Cultural Organization · Intangible Cultural Heritage

Intangible cultural heritage

Connection EN FR ES

Search ICH website

Webpages, elements, decisions...

NEWS | EVENTS | CONVENTION | LISTS | SAFEGUARDING | ACTORS | THEMES | COUNTRY

UNESCO » Culture » Intangible Heritage » Lists » Lacemaking in Croatia

0:00 / 0:12

Nomination file No. 00245

- Nomination form: **English**|**French**
- Consent of communities: **English/Croatian**

Decision

Inscription: **4.COM 13.32**

Periodic reporting

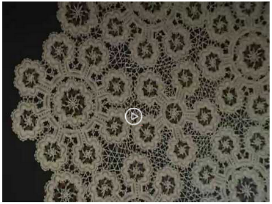
The report on the implementation of the Convention, due by States Parties every 6 years, includes a section on the elements inscribed on the

Lacemaking in Croatia

Croatia

Inscribed in 2009 (**4.COM**) on the Representative List of the Intangible Cultural Heritage of Humanity

At least three distinct traditions of Lacemaking in Croatia persist today, centred on the towns of Pag on the Adriatic, Lepoglava in northern Croatia and Hvar on the Dalmatian island of the same name. Pag needle-point lace was originally used to make ecclesiastical garments, tablecloths and ornaments for clothing. The process involves embellishing a spider web pattern with geometrical motifs and is transmitted today by older women who offer year-long courses. Lepoglava bobbin lace is made by braiding thread wound on spindles, or bobbins; it is often used to make lace ribbons for folk costumes or is sold at village fairs. An International Lace Festival in Lepoglava celebrates the art every year. Aloe lace is made in Croatia only by Benedictine nuns in the town of Hvar. Thin, white threads are obtained from the core of fresh aloe leaves and woven into a net or other pattern on a cardboard background. The resulting pieces are a symbol of Hvar. Each variety of lace has long been created by rural women as a source of additional income and has left a permanent mark on the culture of its region. The craft both produces an important component of traditional clothes and is itself testimony to a living cultural tradition.



© 2008 by Ministry of Culture :

Source: https://ich.unesco.org/en/RL/lacemaking-in-croatia-00245

Combinatorial designs and automorphism groups

Let t, v, k, and λ be integers. A t-(v, k, λ) design is a family of k-subsets $\mathcal{B} \subseteq$`Combinations([1..v],k)` such that every t-subset of `[1..v]` is contained (as a subset) in exactly λ elements of $\mathcal{B}$. The elements of `[1..v]` are called points, and the elements of $\mathcal{B}$ are called blocks.

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gap> ksub:=Combinations([1..7],3);
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7
gap> d:=BlockDesign(7,B);
rec( blocks := [ [ 1, 2, 4 ], [ 1, 3, 7 ], [ 1, 5, 6 ], [ 2, 3, 5 ],
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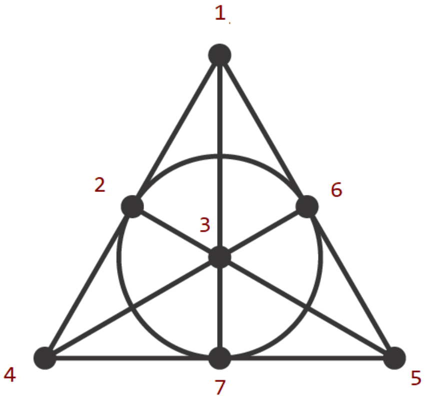
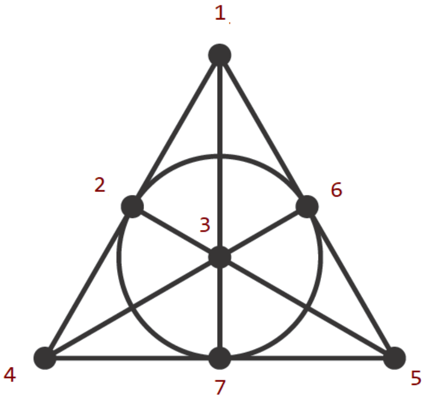
gap> AllTDesignLambdas(d);
[ 7, 3, 1 ]
```

Combinatorial designs and automorphism groups






```
gap> BlocksToIncidenceMat(B);
[ [ 1, 1, 1, 0, 0, 0, 0 ],
  [ 1, 0, 0, 1, 1, 0, 0 ],
  [ 0, 1, 0, 1, 0, 1, 0 ],
  [ 1, 0, 0, 0, 0, 1, 1 ],
  [ 0, 0, 1, 1, 0, 0, 1 ],
  [ 0, 0, 1, 0, 1, 1, 0 ],
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```

Combinatorial designs and automorphism groups

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gap> tsub:=Combinations([1..7],2);
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  [ 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 ],
  [ 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 ],
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  [ 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 ],
  [ 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 ],
  [ 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 ],
  [ 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1 ],
  [ 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1 ],
  [ 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1 ],
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  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1 ],
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 0, 1 ],
  [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 0, 1 ],
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Combinatorial designs and automorphism groups

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          ⋮
 Loops: 418
 Total number of solutions: 30
 total enumeration time: 0:00:00
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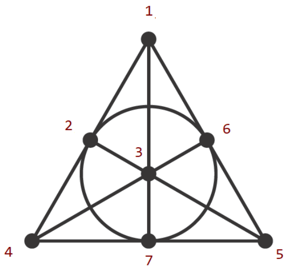

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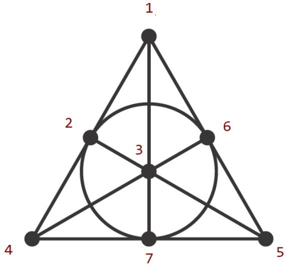
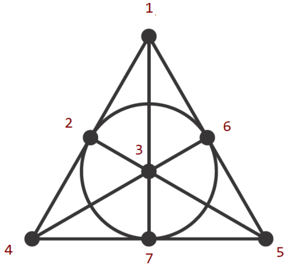
gap> KramerMesnerSearch(2,7,3,1,g);
Computing t-subset orbit representatives...
7
Computing k-subset orbit representatives...
13
Removing k-subset orbits covering t-subset orbits more than lambda times...
11
Computing the Kramer-Mesner matrix...
[ 8, 12 ]
Starting solver...
        ⋮
```



Combinatorial designs and automorphism groups

```
gap> tsub:=Combinations([1..7],2);
[ [ 1, 2 ], [ 1, 3 ], [ 1, 4 ], [ 1, 5 ], [ 1, 6 ], [ 1, 7 ], [ 2, 3 ],
  [ 2, 4 ], [ 2, 5 ], [ 2, 6 ], [ 2, 7 ], [ 3, 4 ], [ 3, 5 ], [ 3, 6 ],
  [ 3, 7 ], [ 4, 5 ], [ 4, 6 ], [ 4, 7 ], [ 5, 6 ], [ 5, 7 ], [ 6, 7 ] ]
gap> Size(tsub);
21
gap> km:=KramerMesnerMat(Group(()),tsub,ksub,1)
gap> sol:=SolveKramerMesner(km);;
        ⋮
 Loops: 418
 Total number of solutions: 30
 total enumeration time: 0:00:00
gap> g:=Group((1,5,4)(2,6,7));;
gap> KramerMesnerSearch(2,7,3,1,g);
        ⋮
 Loops: 37
 Total number of solutions: 6
 total enumeration time: 0:00:00
 Performing isomorph rejection...
 1
 [ rec( autSubgroup := Group([ (1,5,4)(2,6,7) ]),
       blocks := [ [ 1, 2, 5 ], [ 1, 3, 6 ], [ 1, 4, 7 ], [ 2, 3, 4 ],
           [ 2, 6, 7 ], [ 3, 5, 7 ], [ 4, 5, 6 ] ], isBlockDesign := true,
       v := 7 ) ]
```



E. S. Kramer, D. M. Mesner, *t-designs on hypergraphs*, Discrete Math. 15 (1976), no. 3, 263–296.

Combinatorial designs and automorphism groups

E. S. Kramer, D. M. Mesner, *t-designs on hypergraphs*, Discrete Math. 15 (1976), no. 3, 263–296.

M. Kiermaier, V. Krčadinac, V. D. Tonchev, R. Vlahović Kruc, A. Wassermann, *Some new Steiner designs* $S(2, 6, 91)$, J. Algebraic Combin. 62 (2025), no. 2, Paper No. 38, 11 pp.

Combinatorial designs and automorphism groups

E. S. Kramer, D. M. Mesner, *t-designs on hypergraphs*, Discrete Math. 15 (1976), no. 3, 263–296.

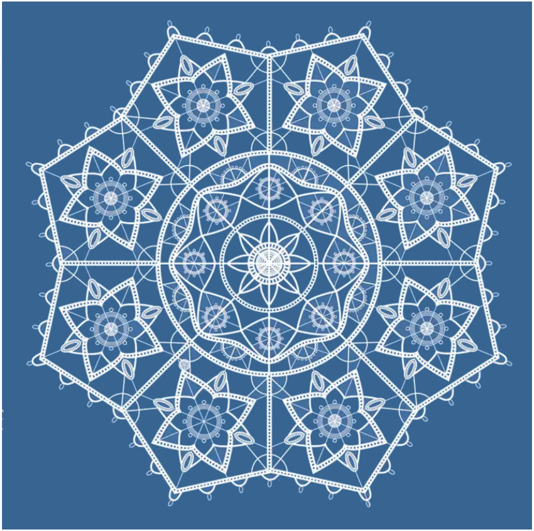
M. Kiermaier, V. Krčadinac, V. D. Tonchev, R. Vlahović Kruc, A. Wassermann, *Some new Steiner designs* $S(2, 6, 91)$, J. Algebraic Combin. 62 (2025), no. 2, Paper No. 38, 11 pp.

C. J. Colbourn, J. H. Dinitz (eds.), *Handbook of combinatorial designs. Second edition*, Chapman & Hall/CRC, Boca Raton, FL, 2007.

No	v	b	r	k	λ	Nd	Nr	Comments, Ref	Where?
47	11	22	10	5	4	4393	-	2#7,D#63 [323]	
48	51	85	10	6	1	?	-		
49	21	30	10	7	3	3809	0	R#54,×3 [1016, 1241, 1946]	
50	36	45	10	8	2	0	-	R#53*,×2	
71	10	30	12	4	4	13769944	-	2#10 [696]	
72	25	60	12	5	2	≥ 118884	≥ 748	2#11 [1225, 2062]	
73	61	122	12	6	1	?	-		
74	31	62	12	6	2	≥ 72	-	2#12 [1239]	

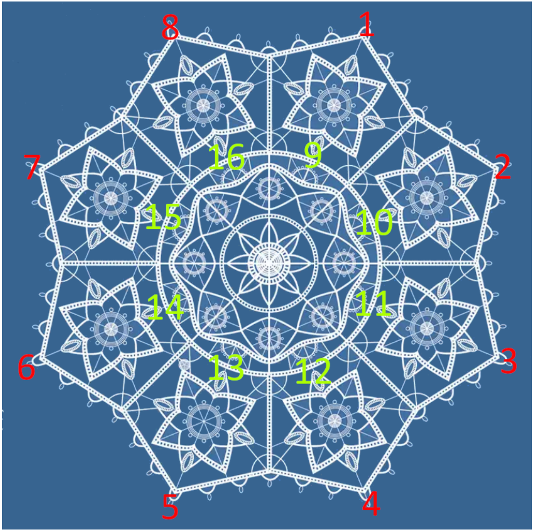
How to choose a group?

How to choose a group?



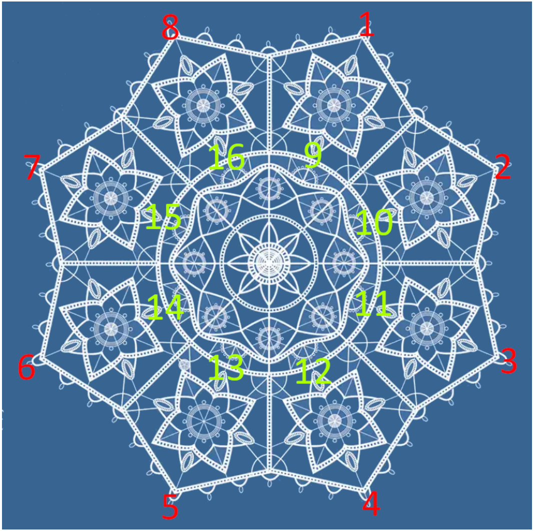
Source: https://www.plakati.com.hr

How to choose a group?



Source: https://www.plakati.com.hr

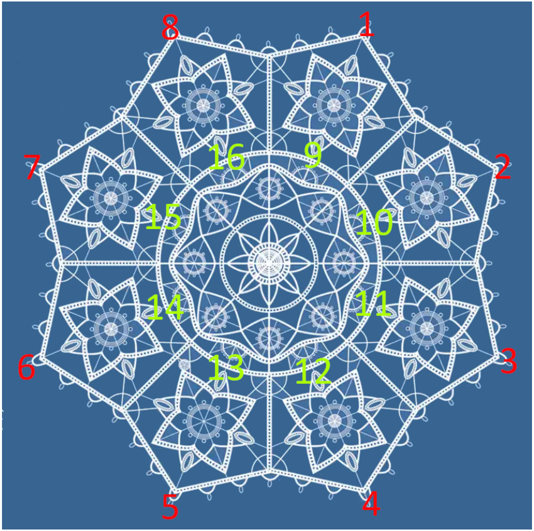
How to choose a group?



Source: https://www.plakati.com.hr

```
gap> a:=(1,2,3,4,5,6,7,8)
> (9,10,11,12,13,14,15,16);;
```

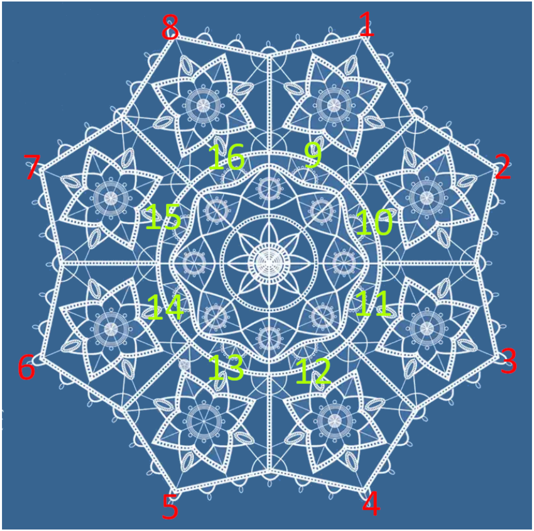
How to choose a group?



Source: https://www.plakati.com.hr

```
gap> a:=(1,2,3,4,5,6,7,8)
> (9,10,11,12,13,14,15,16);;
gap> b:=(1,8)(2,7)(3,6)(4,5)
> (9,16)(10,15)(11,14)(12,13);;
```

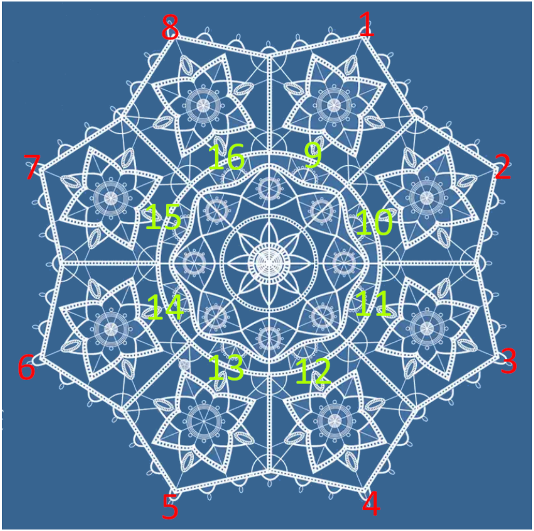
How to choose a group?



Source: https://www.plakati.com.hr

```
gap> a:=(1,2,3,4,5,6,7,8)
> (9,10,11,12,13,14,15,16);;
gap> b:=(1,8)(2,7)(3,6)(4,5)
> (9,16)(10,15)(11,14)(12,13);;
gap> g:=Group(a,b);;
```

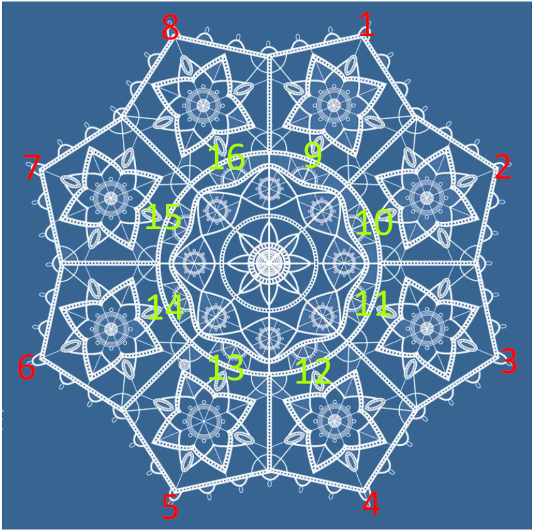
How to choose a group?



Source: https://www.plakati.com.hr

```
gap> a:=(1,2,3,4,5,6,7,8)
> (9,10,11,12,13,14,15,16);;
gap> b:=(1,8)(2,7)(3,6)(4,5)
> (9,16)(10,15)(11,14)(12,13);;
gap> g:=Group(a,b);;
gap> StructureDescription(g);
"D16"
```

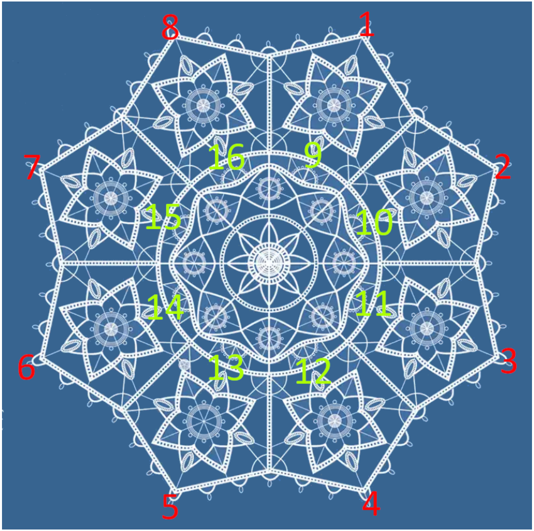
How to choose a group?



Source: https://www.plakati.com.hr

```
gap> a:=(1,2,3,4,5,6,7,8)
> (9,10,11,12,13,14,15,16);;
gap> b:=(1,8)(2,7)(3,6)(4,5)
> (9,16)(10,15)(11,14)(12,13);;
gap> g:=Group(a,b);;
gap> StructureDescription(g);
"D16"
gap> c:=(1,9)(2,10)(3,11)(4,12)
> (5,13)(6,14)(7,15)(8,16);;
```

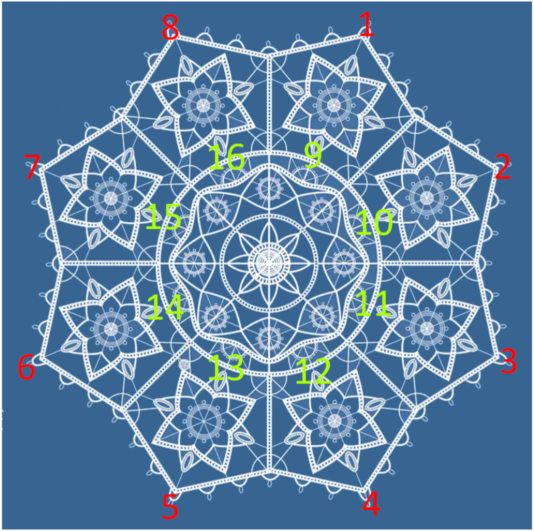
How to choose a group?



Source: https://www.plakati.com.hr

```
gap> a:=(1,2,3,4,5,6,7,8)
> (9,10,11,12,13,14,15,16);;
gap> b:=(1,8)(2,7)(3,6)(4,5)
> (9,16)(10,15)(11,14)(12,13);;
gap> g:=Group(a,b);;
gap> StructureDescription(g);
"D16"
gap> c:=(1,9)(2,10)(3,11)(4,12)
> (5,13)(6,14)(7,15)(8,16);;
gap> h:=Group(a,b,c);;
```

How to choose a group?



Source: https://www.plakati.com.hr

```
gap> a:=(1,2,3,4,5,6,7,8)
> (9,10,11,12,13,14,15,16);;
gap> b:=(1,8)(2,7)(3,6)(4,5)
> (9,16)(10,15)(11,14)(12,13);;
gap> g:=Group(a,b);;
gap> StructureDescription(g);
"D16"
gap> c:=(1,9)(2,10)(3,11)(4,12)
> (5,13)(6,14)(7,15)(8,16);;
gap> h:=Group(a,b,c);;
gap> StructureDescription(h);
"C2 x D16"
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,4,1,g);;
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,4,1,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
136
 .
 .
 .
Total number of solutions: 0
Performing isomorph rejection...
0
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,4,1,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
136
 .
 .
 .
Total number of solutions: 0
Performing isomorph rejection...
0
```

Number of 2-$(16, 4, 1)$ designs: **1**
(affine plane of order 4)

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,4,1,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
136
 .
 .
 .
Total number of solutions: 0
Performing isomorph rejection...
0
```

Number of 2-$(16, 4, 1)$ designs: **1**
(affine plane of order 4)

```
gap> KramerMesnerSearch(3,16,4,1,g);;
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,4,1,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
136
 ⋮
Total number of solutions: 0
Performing isomorph rejection...
0
```

Number of 2-$(16, 4, 1)$ designs: **1** (affine plane of order 4)

```
gap> KramerMesnerSearch(3,16,4,1,g);;
Computing t-subset orbit representatives...
42
Computing k-subset orbit representatives...
136
 ⋮
Total number of solutions: 152
Performing isomorph rejection...
30
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,4,1,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
136
 ⋮
Total number of solutions: 0
Performing isomorph rejection...
0
```

Number of 2-$(16, 4, 1)$ designs: **1** (affine plane of order 4)

```
gap> KramerMesnerSearch(3,16,4,1,g);;
Computing t-subset orbit representatives...
42
Computing k-subset orbit representatives...
136
 ⋮
Total number of solutions: 152
Performing isomorph rejection...
30
```

Number of 3-$(16, 4, 1)$ designs or SQS(16): **1 054 163**

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,6,2,g);;
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,6,2,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
547
 .
 .
 .
Total number of solutions: 8
Performing isomorph rejection...
2
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,6,2,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
547
 .
 .
 .
Total number of solutions: 8
Performing isomorph rejection...
2
```

Number of 2-$(16, 6, 2)$ designs: **3**
(biplanes, i.e. SBIBDs with $\lambda = 2$)

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,6,2,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
547
 .
 .
 .
Total number of solutions: 8
Performing isomorph rejection...
2
```

Number of 2-$(16, 6, 2)$ designs: **3**
(biplanes, i.e. SBIBDs with $\lambda = 2$)

```
gap> KramerMesnerSearch(5,16,6,3,h,rec(StopAfter:=100));;
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,6,2,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
547
 ⋮
Total number of solutions: 8
Performing isomorph rejection...
2
```

Number of 2-$(16,6,2)$ designs: **3**
(biplanes, i.e. SBIBDs with $\lambda = 2$)

```
gap> KramerMesnerSearch(5,16,6,3,h,rec(StopAfter:=100));;
Computing t-subset orbit representatives...
147
Computing k-subset orbit representatives...
291
 ⋮
Stopped after number of solutions: 100
Performing isomorph rejection...
100
```

How to construct designs in PAG?

```
gap> KramerMesnerSearch(2,16,6,2,g);;
Computing t-subset orbit representatives...
13
Computing k-subset orbit representatives...
547
 ⋮
Total number of solutions: 8
Performing isomorph rejection...
2
```

Number of 2-$(16,6,2)$ designs: **3**
(biplanes, i.e. SBIBDs with $\lambda = 2$)

```
gap> KramerMesnerSearch(5,16,6,3,h,rec(StopAfter:=100));;
Computing t-subset orbit representatives...
147
Computing k-subset orbit representatives...
291
 ⋮
Stopped after number of solutions: 100
Performing isomorph rejection...
100
```

Number of 5-$(16,6,3)$ designs: **?**

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

An existence result: $3\text{-}(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> ?DerivedBlockDesign
```

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> ?DerivedBlockDesign
Help: Showing `DESIGN: DerivedBlockDesign'

> DerivedBlockDesign( <D>, <x> )

Suppose  <D>  is  a block design, and <x> is a point or block of <D>.
Then  this  function  returns  the *derived design* $DD$ of <D>, with
respect to <x>.

If  <x>  is  a  point  then $DD$ is the block design whose blocks are
those  of <D> containing <x>, but with <x> deleted from these blocks,
and the points of $DD$ are those which occur in some block of $DD$.
```

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> ?DerivedBlockDesign
Help: Showing `DESIGN: DerivedBlockDesign'

> DerivedBlockDesign( <D>, <x> )

Suppose  <D>  is  a block design, and <x> is a point or block of <D>.
Then  this  function  returns  the *derived design* $DD$ of <D>, with
respect to <x>.

If  <x>  is  a  point  then $DD$ is the block design whose blocks are
those  of <D> containing <x>, but with <x> deleted from these blocks,
and the points of $DD$ are those which occur in some block of $DD$.
```

$$3\text{-}(42, 6, 1) \quad \rightsquigarrow \quad 2\text{-}(41, 5, 1)$$

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> ?ExtendSteinerDesign
```

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> ?ExtendSteinerDesign
Help: Showing `PAG: ExtendSteinerDesign'
  2.3-4 ExtendSteinerDesign

  â£ ExtendSteinerDesign( d[, opt] ) ââââââââââââââââââââââââ function

  Performs a search for extensions of a Steiner design d. By default,
  the  extended  designs  are  assumed to be invariant under the full
  automorphism  group of d. This can be changed by setting options in
  the record opt.
```

An existence result: 3-(42, 6, 1) designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> ?ExtendSteinerDesign
Help: Showing `PAG: ExtendSteinerDesign'
  2.3-4 ExtendSteinerDesign

  â£ ExtendSteinerDesign( d[, opt] ) ââââââââââââââââââââââââ function

  Performs a search for extensions of a Steiner design d. By default,
  the  extended  designs  are  assumed to be invariant under the full
  automorphism  group of d. This can be changed by setting options in
  the record opt.
```

$$2\text{-}(41, 5, 1) \rightsquigarrow 3\text{-}(42, 6, 1)$$

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> ?ExtendSteinerDesign
Help: Showing `PAG: ExtendSteinerDesign'
  2.3-4 ExtendSteinerDesign

  â£ ExtendSteinerDesign( d[, opt] ) ââââââââââââââââââââââââ function

  Performs a search for extensions of a Steiner design d. By default,
  the  extended  designs  are  assumed to be invariant under the full
  automorphism  group of d. This can be changed by setting options in
  the record opt.
```

$$2\text{-}(41,5,1) \quad \rightsquigarrow \quad 3\text{-}(42,6,1)$$

V. Krčadinac, *Steiner 2-designs* $S(2,5,41)$ *with automorphisms of order* 3, J. Combin. Math. Combin. Comput. 43 (2002), 83–99.

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

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Small Steiner 3-designs

Kiermaier, Michael[1]; Krčadinac, Vedran[2]; Wassermann, Alfred[1]

Show affiliations

This dataset contains the known Steiner 3-(v,k,1) designs with v<=50 and their extensions to Steiner 5-designs. Rotational Steiner quadruple systems of orders v=46 and v=92 are also included, as well as Steiner 2-designs with parameters 2-(40,4,1), 2-(41,5,1), 2-(45,5,1), and 2-(49,4,1). The designs were compiled in the study

- M. Kiermaier, V. Krčadinac, A. Wassermann, "*Steiner 3-designs as extensions*", preprint, 2025, https://arxiv.org/abs/2509.23483.

The files are gnu-zipped text files containig designs in a format compatible with the computer algebra system GAP and the DESIGN package. Each file named t-v-k-lambda.des.gz contains a list of t-(v,k,lambda) designs. An on-line table of the designs in this dataset is also available here.

Files

Files (915.4 MB)

Name	Size	Download all
2-40-4-1.des.gz md5:fd1f5cd242fedfc7fe9aac6643542ce6	1.4 MB	Download
2-41-5-1.des.gz md5:3e30df9a4e6a210ceaba7d8a1823c099	7.2 kB	Download

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> Read("2-41-5-1.des");
```

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> Read("2-41-5-1.des");
gap> Size(d);
15
```

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> Read("2-41-5-1.des");
gap> Size(d);
15
gap> aut:=List(d,BlockDesignAut);;
```

An existence result: 3-$(42, 6, 1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> Read("2-41-5-1.des");
gap> Size(d);
15
gap> aut:=List(d,BlockDesignAut);;
gap> List(aut,Size);
[ 205, 120, 120, 24, 24, 20, 18, 18, 18, 18, 12, 9, 6, 6, 2 ]
```

An existence result: 3-$(42,6,1)$ designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> Read("2-41-5-1.des");
gap> Size(d);
15
gap> aut:=List(d,BlockDesignAut);;
gap> List(aut,Size);
[ 205, 120, 120, 24, 24, 20, 18, 18, 18, 18, 12, 9, 6, 6, 2 ]
gap> e:=ExtendSteinerDesign(d[4]);;
Extending 2-(41,5,1) design with group of order 24
(4841 options, 432+0 items, 98879 entries successfully read)
Altogether 0 solutions, 1781664+906 mems, 0 updates, 0 cleansings,...
```

An existence result: 3-(42, 6, 1) designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> Read("2-41-5-1.des");
gap> Size(d);
15
gap> aut:=List(d,BlockDesignAut);;
gap> List(aut,Size);
[ 205, 120, 120, 24, 24, 20, 18, 18, 18, 18, 12, 9, 6, 6, 2 ]
gap> e:=ExtendSteinerDesign(d[4]);;
Extending 2-(41,5,1) design with group of order 24
(4841 options, 432+0 items, 98879 entries successfully read)
Altogether 0 solutions, 1781664+906 mems, 0 updates, 0 cleansings,...
gap> e:=ExtendSteinerDesign(d[5]);;
Extending 2-(41,5,1) design with group of order 24
(10832 options, 414+0 items, 227014 entries successfully read)
Altogether 1 solution, 3940647+470871118 mems, 38143494 updates,...
```

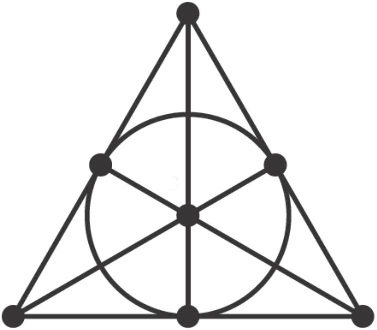
An existence result: 3-(42, 6, 1) designs

M. Kiermaier, V. Krčadinac, A. Wassermann, *Steiner 3-designs as extensions*, preprint, 2025. https://arxiv.org/abs/2509.23483

```
gap> Read("2-41-5-1.des");
gap> Size(d);
15
gap> aut:=List(d,BlockDesignAut);;
gap> List(aut,Size);
[ 205, 120, 120, 24, 24, 20, 18, 18, 18, 18, 12, 9, 6, 6, 2 ]
gap> e:=ExtendSteinerDesign(d[4]);;
Extending 2-(41,5,1) design with group of order 24
(4841 options, 432+0 items, 98879 entries successfully read)
Altogether 0 solutions, 1781664+906 mems, 0 updates, 0 cleansings,...
gap> e:=ExtendSteinerDesign(d[5]);;
Extending 2-(41,5,1) design with group of order 24
(10832 options, 414+0 items, 227014 entries successfully read)
Altogether 1 solution, 3940647+470871118 mems, 38143494 updates,...
gap> List(e,AllTDesignLambdas);
[ [ 574, 82, 10, 1 ] ]
```

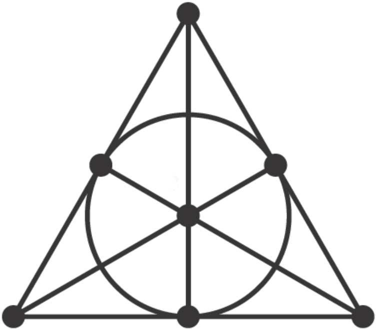
Contents

Mosaics of combinatorial designs



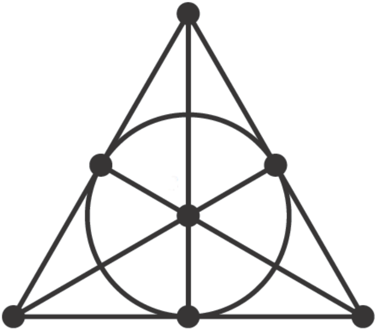
```
[ [ 1, 1, 1, 0, 0, 0, 0 ],
  [ 1, 0, 0, 1, 1, 0, 0 ],
  [ 0, 1, 0, 1, 0, 1, 0 ],
  [ 1, 0, 0, 0, 0, 1, 1 ],
  [ 0, 0, 1, 1, 0, 0, 1 ],
  [ 0, 0, 1, 0, 1, 1, 0 ],
  [ 0, 1, 0, 0, 1, 0, 1 ] ]
```

Mosaics of combinatorial designs



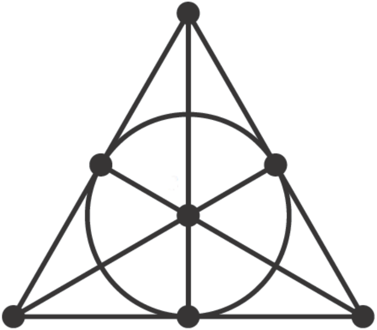
```
[ [ 0, 1, 1, 0, 1, 0, 0 ],
  [ 0, 0, 1, 1, 0, 1, 0 ],
  [ 0, 0, 0, 1, 1, 0, 1 ],
  [ 1, 0, 0, 0, 1, 1, 0 ],
  [ 0, 1, 0, 0, 0, 1, 1 ],
  [ 1, 0, 1, 0, 0, 0, 1 ],
  [ 1, 1, 0, 1, 0, 0, 0 ] ]
```

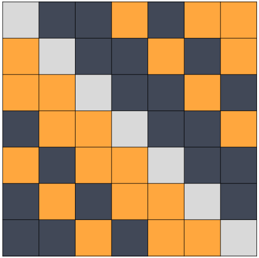
Mosaics of combinatorial designs



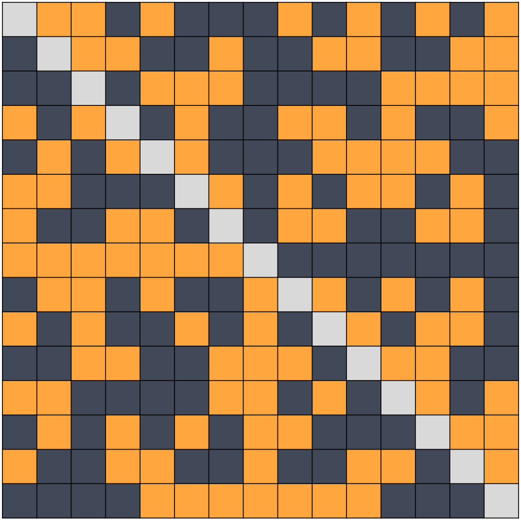
```
[ [ 0, 1, 1, 2, 1, 2, 2 ],
  [ 2, 0, 1, 1, 2, 1, 2 ],
  [ 2, 2, 0, 1, 1, 2, 1 ],
  [ 1, 2, 2, 0, 1, 1, 2 ],
  [ 2, 1, 2, 2, 0, 1, 1 ],
  [ 1, 2, 1, 2, 2, 0, 1 ],
  [ 1, 1, 2, 1, 2, 2, 0 ] ]
```

Mosaics of combinatorial designs

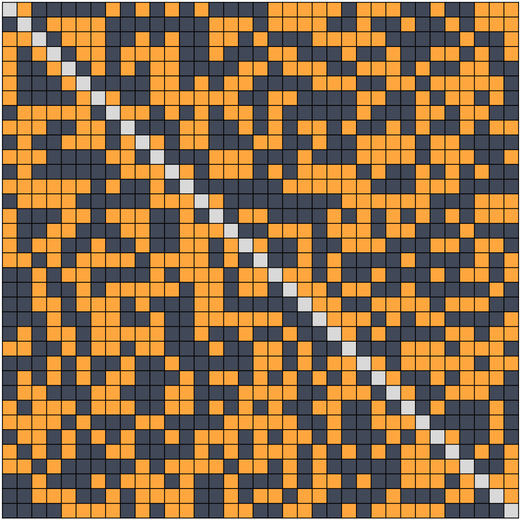




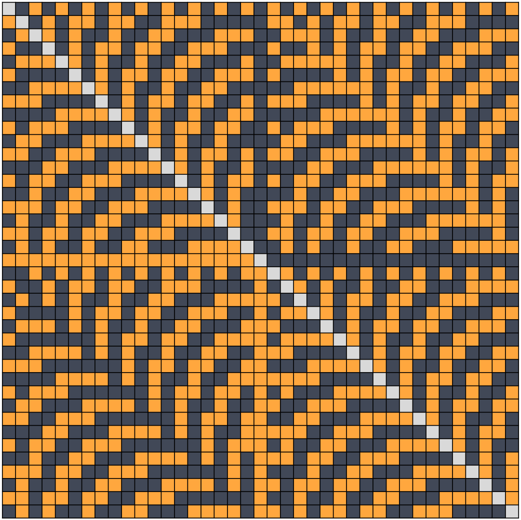
Skew Hadamard mosaics



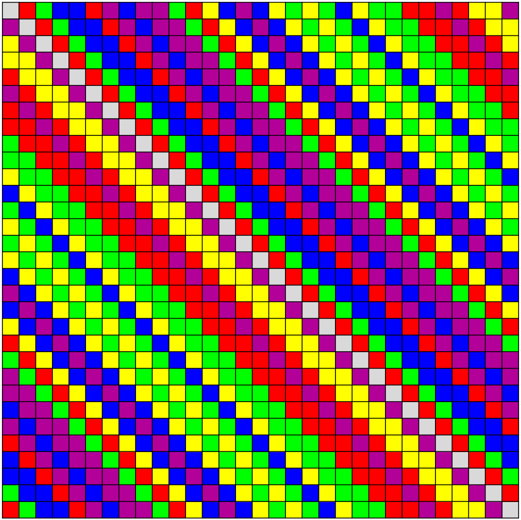
Skew Hadamard mosaics



Skew Hadamard mosaics



A mosaic of projective planes of order 5



Difference sets ⇝ tilings ⇝ mosaics

```
gap> ?DifSets: Definitions
Help: Showing `DifSets: Definitions'

  1 Definitions


  1.1 Difference Sets

  A  ( v,  k, λ )-difference set is a nonempty proper subset D of a finite group G such
  that  |G|  = v, |D| = k, and each nonidentity element of G can be written as d_id_j^-1
  for  d_i,  d_j  â  D  in exactly λ different ways. The standard example is the ( 7, 3,
  1)-difference  set  {1,  2,  4} of the group Z/7Z under addition. Additionally, it can
  easily  be shown that every one element subset of a group is a difference set, and the
  complement of any difference set is also a difference set.
```

Difference sets ⇝ tilings ⇝ mosaics

```
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  1 Definitions


  1.1 Difference Sets

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  complement of any difference set is also a difference set.
gap> t:=[ [ 2, 6, 12, 25, 26, 28 ], [ 3, 11, 18, 20, 23, 24 ], [ 4, 5, 8, 14, 16, 21 ],
> [ 7, 9, 10, 15, 27, 31 ], [ 13, 17, 19, 22, 29, 30 ] ];;
```

Difference sets ⇝ tilings ⇝ mosaics

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> [ 7, 9, 10, 15, 27, 31 ], [ 13, 17, 19, 22, 29, 30 ] ];;
gap> m:=DifferenceMosaic(CyclicGroup(31), t);
[ [ 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2, 1, 1, 4, 1, 5, 5, 4 ],
  [ 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2, 1, 1, 4, 1, 5, 5 ],
  [ 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2, 1, 1, 4, 1, 5 ],
  [ 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2, 1, 1, 4, 1 ],
  [ 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2, 1, 1, 4 ],
  [ 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2, 1, 1 ],
  [ 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2, 1 ],
  [ 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2, 2 ],
  [ 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5, 2 ],
  [ 2, 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3, 5 ],
  [ 5, 2, 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2, 3 ],
  [ 3, 5, 2, 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5, 2 ],
  [ 2, 3, 5, 2, 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2, 5 ],
  [ 5, 2, 3, 5, 2, 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5, 2 ],
  [ 2, 5, 2, 3, 5, 2, 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3, 5 ],
  [ 5, 2, 5, 2, 3, 5, 2, 2, 1, 1, 4, 1, 5, 5, 4, 0, 1, 2, 3, 3, 1, 4, 3, 4, 4, 2, 1, 5, 3, 4, 3 ],
```

Difference sets ⇝ tilings ⇝ mosaics

```
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  1 Definitions


  1.1 Difference Sets

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gap> m:=DifferenceMosaic(CyclicGroup(31), t);;
```

Difference sets ⇝ tilings ⇝ mosaics

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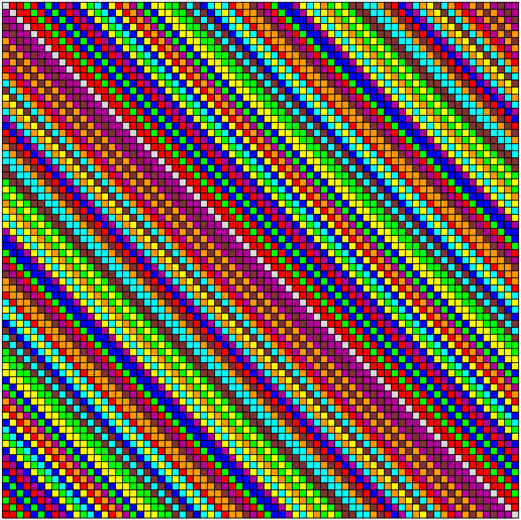
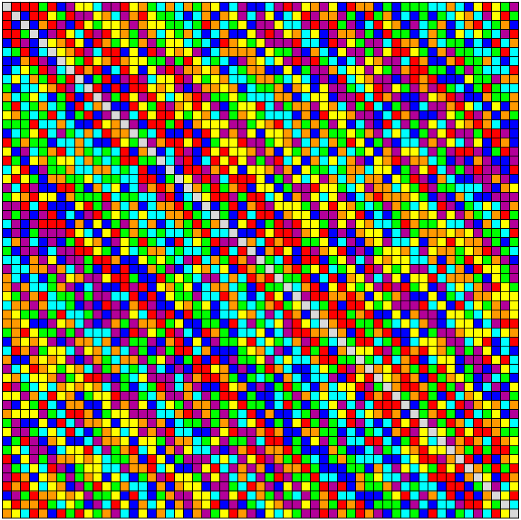
> [ 7, 9, 10, 15, 27, 31 ], [ 13, 17, 19, 22, 29, 30 ] ];;
gap> m:=DifferenceMosaic(CyclicGroup(31), t);;
gap> MosaicParameters(m);
"2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1) + 2-(31,6,1)"
```

A mosaic of projective planes of order 8



A mosaic of projective planes of order 7



General mosaics

A. Ćustić, V. Krčadinac, Y. Zhou, *Tiling groups with difference sets*, Electron. J. Combin. **22** (2015), no. 2, Paper 2.56, 13 pp.

General mosaics

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O. W. Gnilke, M. Greferath, M. O. Pavčević, *Mosaics of combinatorial designs*, Des. Codes Cryptogr. **86** (2018), no. 1, 85–95.

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K. Matsubara, M. Sawa, D. Matsumoto, H. Kiyama, S. Kageyama, *An addition structure on incidence matrices of a BIB design*, Ars Combin. **78** (2006), 113–122.

A. Bonnecaze, E. Rains, P. Solé, 3-*colored* 5-*designs and* $\mathbb{Z}_4$-*codes*, J. Statist. Plann. Inference **86** (2000), no. 2, 349–368.

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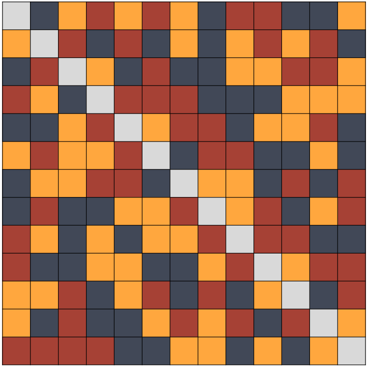
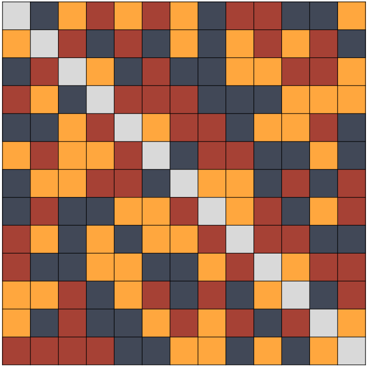
V. Krčadinac, *Small examples of mosaics of combinatorial designs*, Examples and Counterexamples **6** (2024), 100163.

General mosaics



General mosaics



Order	2	3	4	5	7	8	9	...
Tiling	✓	✗	✗	✓	✓	✓	✗	...
Mosaic	✓	✓	?	✓	✓	✓	?	...

Cubes of symmetric designs

```
gap> g:=CyclicGroup(7);;
```

Cubes of symmetric designs

```
gap> g:=CyclicGroup(7);;
gap> ds:=[1,2,4];;
```

Cubes of symmetric designs

```
gap> g:=CyclicGroup(7);;
gap> ds:=[1,2,4];;
gap> IsDifferenceSet(g,ds);
true
```

Cubes of symmetric designs

```
gap> g:=CyclicGroup(7);;
gap> ds:=[1,2,4];;
gap> IsDifferenceSet(g,ds);
true
gap> DifferenceCube(g,ds,3);
[ [ [ 1, 1, 0, 1, 0, 0, 0 ],
      [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ] ],
  [ [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
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      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ],
      [ 1, 1, 0, 1, 0, 0, 0 ] ],
  [ [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
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```

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true
gap> DifferenceCube(g,ds,3);
[ [ [ 1, 1, 0, 1, 0, 0, 0 ],
      [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ] ],
  [ [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ],
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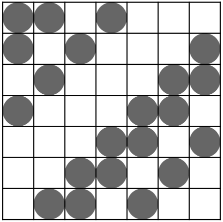

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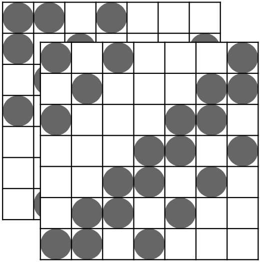
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ] ],
  [ [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ],
      [ 1, 1, 0, 1, 0, 0, 0 ] ],
  [ [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ]
```



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      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ] ],
  [ [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ],
      [ 1, 1, 0, 1, 0, 0, 0 ] ],
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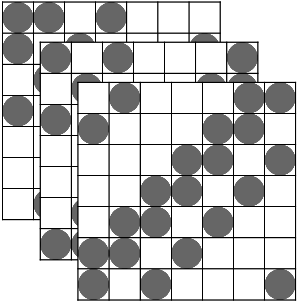
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ]
```



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true
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[ [ [ 1, 1, 0, 1, 0, 0, 0 ],
      [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
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      [ 0, 1, 1, 0, 1, 0, 0 ] ],
  [ [ 1, 0, 1, 0, 0, 0, 1 ],
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      [ 1, 0, 0, 0, 1, 1, 0 ],
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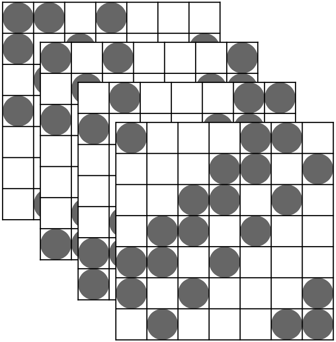
      [ 0, 1, 1, 0, 1, 0, 0 ],
      [ 1, 1, 0, 1, 0, 0, 0 ] ],
  [ [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ]
```



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      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
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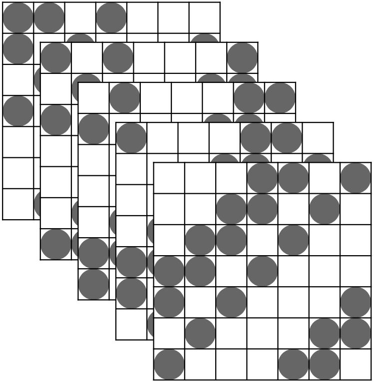
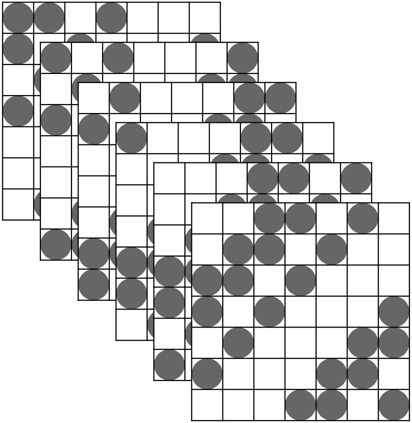
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ],
      [ 1, 1, 0, 1, 0, 0, 0 ] ],
  [ [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ]
```



Cubes of symmetric designs

```
gap> g:=CyclicGroup(7);;
gap> ds:=[1,2,4];;
gap> IsDifferenceSet(g,ds);
true
gap> DifferenceCube(g,ds,3);
[ [ [ 1, 1, 0, 1, 0, 0, 0 ],
      [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ] ],
  [ [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ],
      [ 1, 1, 0, 1, 0, 0, 0 ] ],
  [ [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1
```



Cubes of symmetric designs

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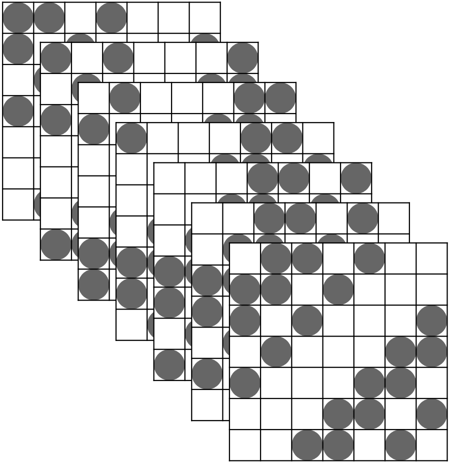
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ] ],
  [ [ 1, 0, 1, 0, 0, 0, 1 ],
      [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ],
      [ 0, 0, 1, 1, 0, 1, 0 ],
      [ 0, 1, 1, 0, 1, 0, 0 ],
      [ 1, 1, 0, 1, 0, 0, 0 ] ],
  [ [ 0, 1, 0, 0, 0, 1, 1 ],
      [ 1, 0, 0, 0, 1, 1, 0 ],
      [ 0, 0, 0, 1, 1, 0, 1 ]
```



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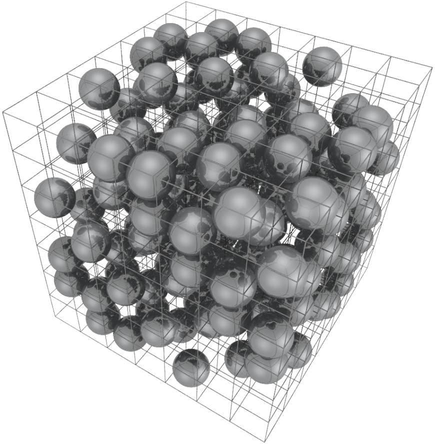

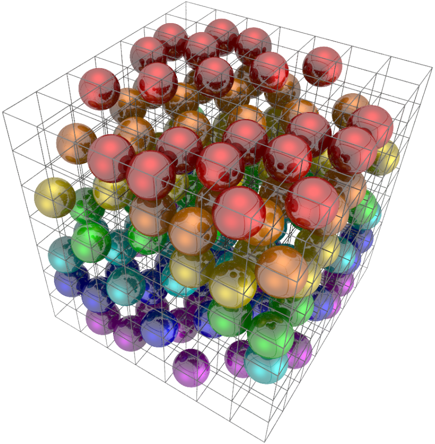

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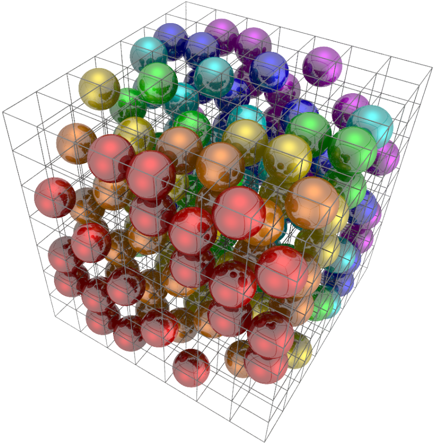

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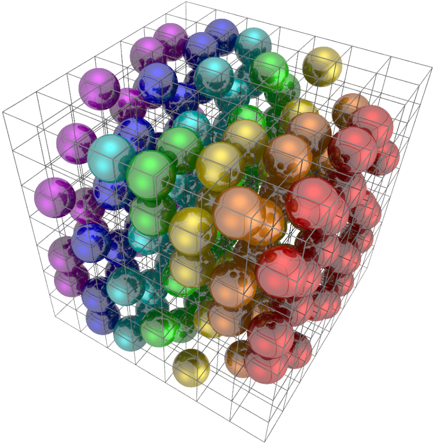

Cubes of symmetric designs

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There are exactly three symmetric 2-$(16, 6, 2)$ designs:

$$|\operatorname{Aut}(\mathcal{D}_1)| = 11520, \quad |\operatorname{Aut}(\mathcal{D}_2)| = 768, \quad |\operatorname{Aut}(\mathcal{D}_3)| = 384$$

Red design, Green design, Blue design

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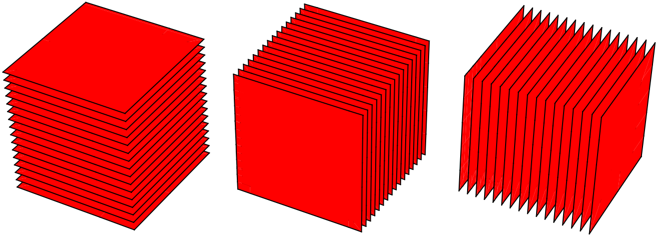
$(16, 6, 2)$ difference set in C_2^4:



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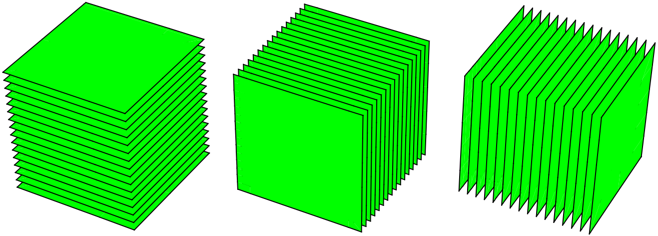
$(16, 6, 2)$ difference set in $C_2 \times C_8$:



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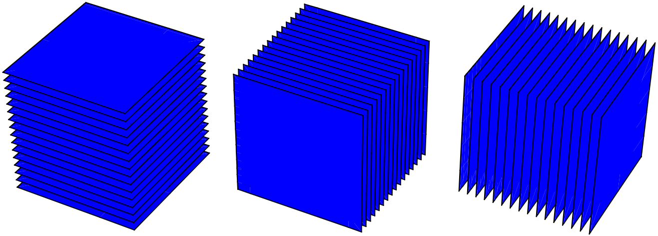
$(16, 6, 2)$ difference set in $C_2 \times Q_8$:



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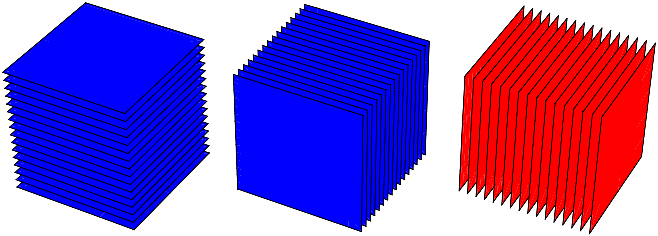

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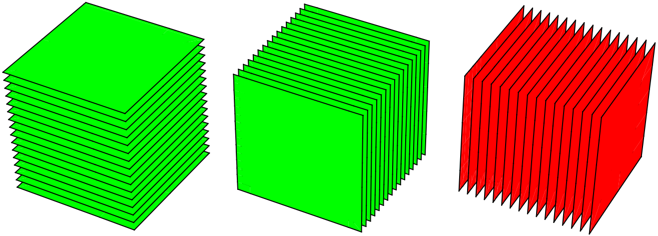

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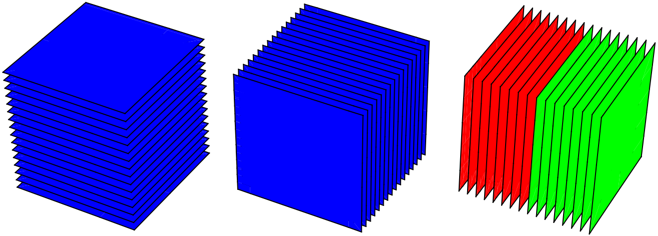

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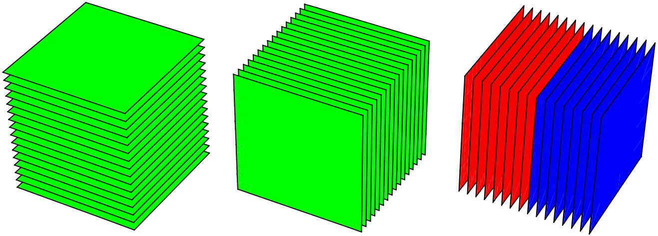

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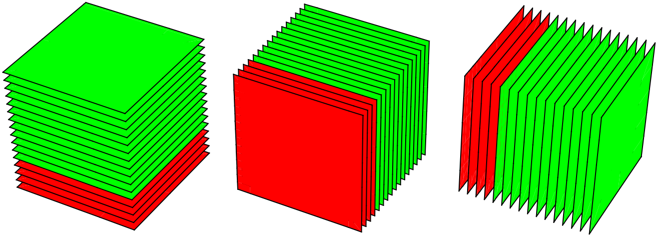

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Theorem (V.K., M.O.Pavčević, K.Tabak)

For every $m \geq 2$ and $n \geq 3$, there are n-cubes of symmetric

$$(4^m,\ 2^{m-1}(2^m - 1),\ 2^{m-1}(2^{m-1} - 1))$$

designs that are not difference cubes.

Projection cubes of symmetric designs

V. Krčadinac, L. Relić, *Projection cubes of symmetric designs*, Math. Comput. Sci. 19 (2025), no. 1, Paper No. 11, 17 pp.

V. Krčadinac, M. O. Pavčević, *On higher-dimensional symmetric designs*, to appear in Exp. Math. (2026). https://arxiv.org/abs/2412.09067

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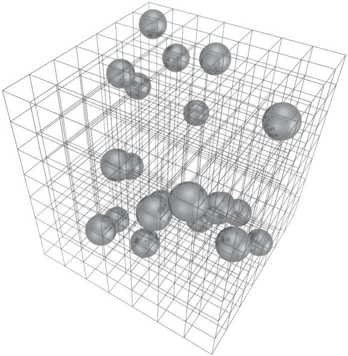

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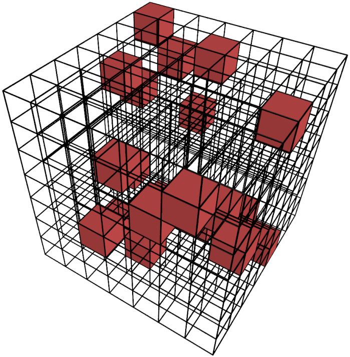

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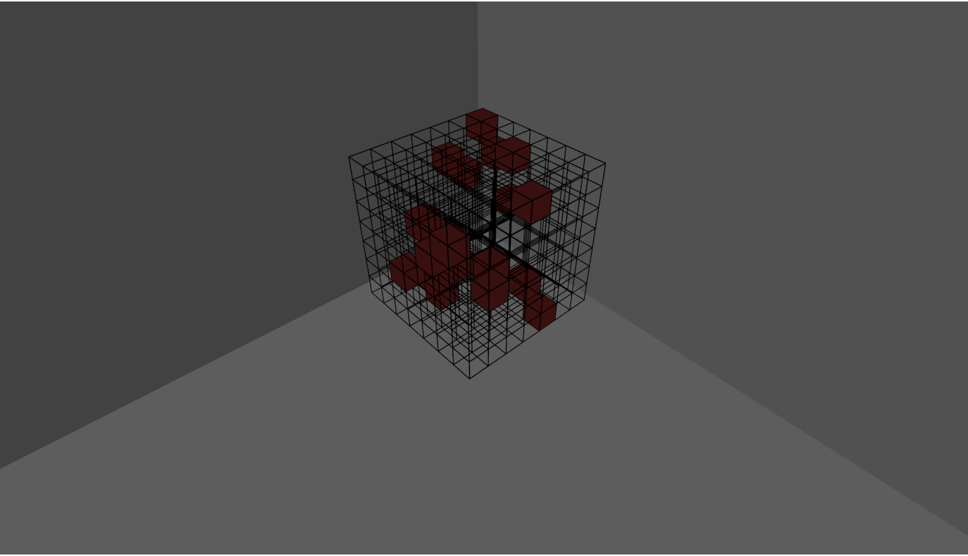

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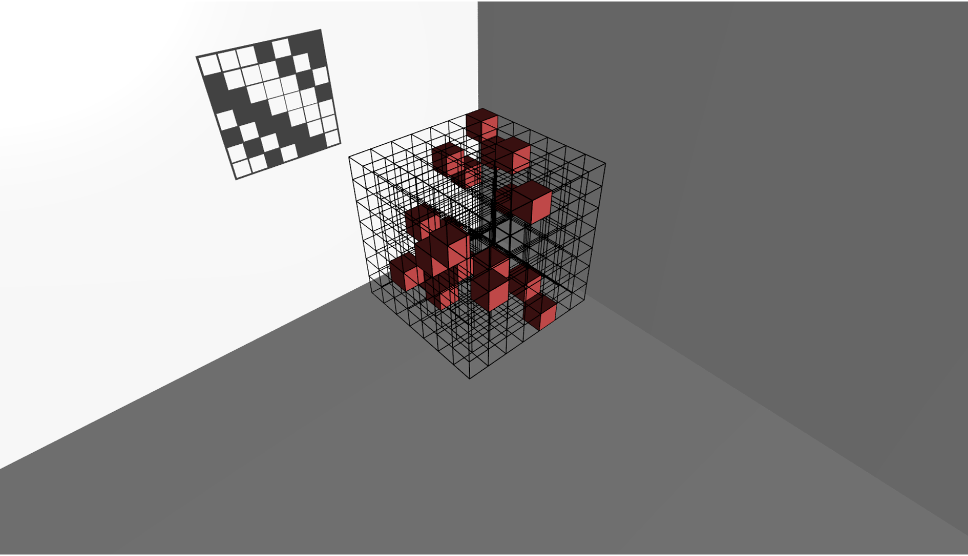

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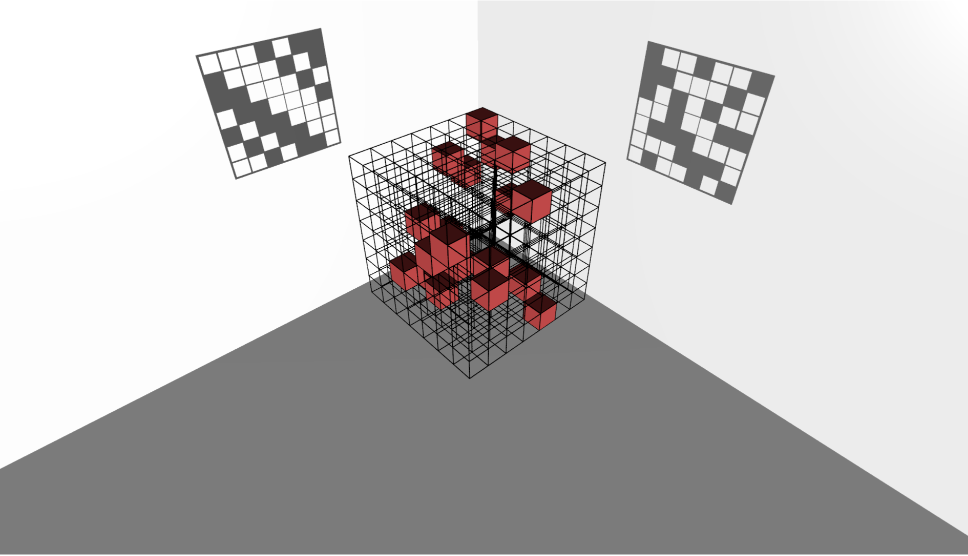

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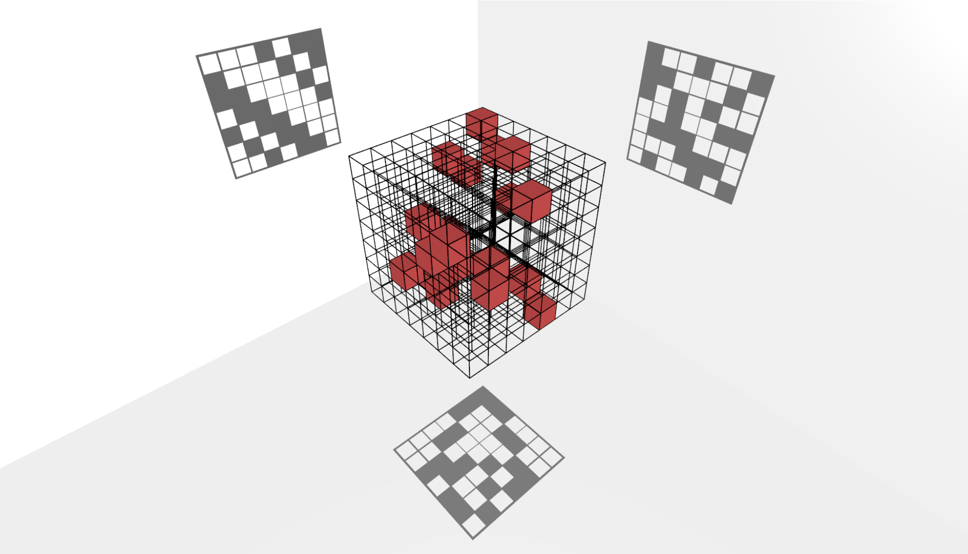

Projection cubes of symmetric designs

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Theorem (V.K., L. Relić)

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An *n-dimensional (v, k, λ) difference set* in G is a set of n-tuples $D \subseteq G^n$ of size $|D| = k$ such that $\{d_i - d_j \mid d \in D\} \subseteq G$ are (v, k, λ) difference sets, for all $1 \leq i < j \leq n$.

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Theorem (V.K., L. Relić)

If D is an n-dimensional (v, k, λ) difference set in G, then the development

$$\mathrm{dev}\, D = \{(d_1 + g, \ldots, d_n + g) \mid g \in G,\ d \in D\} = \mathrm{supp}\, C$$

supports a projection cube $C \in \mathcal{P}^n(v, k, \lambda)$.

Projection cubes of symmetric designs

```
gap> ?PaleyDifferenceSet
Help: Showing `PAG: PaleyDifferenceSet'
  2.7-8 PaleyDifferenceSet

  â£ PaleyDifferenceSet( q ) ââââââââââââââââââââââââââââââ function

  Returns the q-dimensional Paley difference set in GF(q). This is
  a  (q,(q-1)/2,(q-3)/4)  difference  set in the additive group of
  GF(q). See [KR24] for more details.
```

Projection cubes of symmetric designs

```
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gap> ?TwinPrimePowerDifferenceSet
Help: Showing `PAG: TwinPrimePowerDifferenceSet'
  2.7-10 TwinPrimePowerDifferenceSet

  â£ TwinPrimePowerDifferenceSet( q ) ââââââââââââââââââââ function

  Returns  the  q-dimensional twin prime power difference set. For
  n=(q+1)^2/4,  this  is  a  (4n-1,2n-1,n-1) difference set in the
  direct  product GF(q)Ã GF(q+2). Both q and q+2 must be powers of
  primes. See [KR24] for more details.
```

Projection cubes of symmetric designs

```
gap> ds:=PaleyDifferenceSet(7);
[ [ 0*Z(7), Z(7)^0, Z(7), Z(7)^2, Z(7)^3, Z(7)^4, Z(7)^5 ],
  [ 0*Z(7), Z(7)^2, Z(7)^3, Z(7)^4, Z(7)^5, Z(7)^0, Z(7) ],
  [ 0*Z(7), Z(7)^4, Z(7)^5, Z(7)^0, Z(7), Z(7)^2, Z(7)^3 ] ]
```

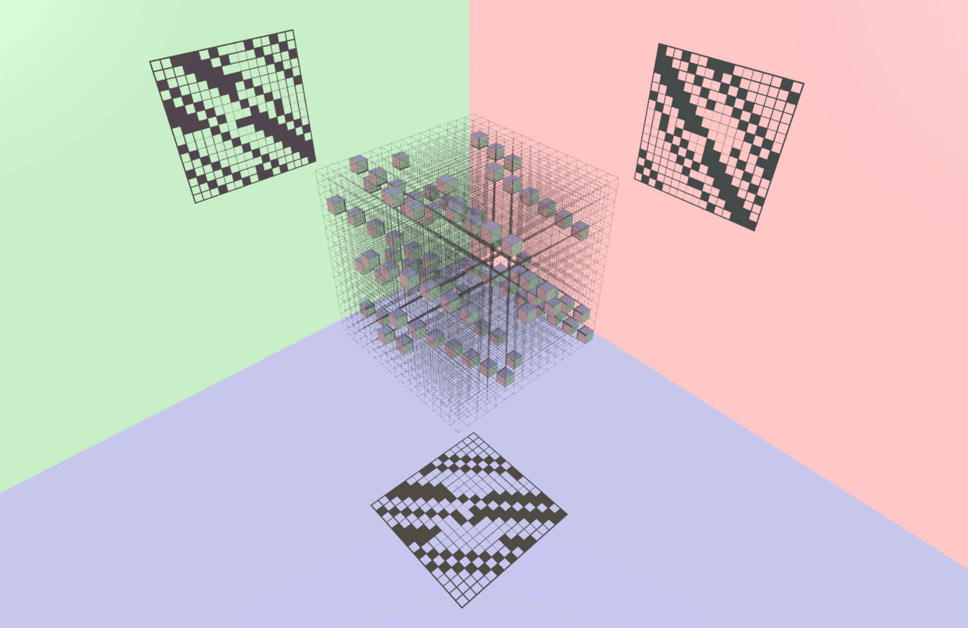
Projection cubes of symmetric designs

```
gap> ds:=PaleyDifferenceSet(7);
[ [ 0*Z(7), Z(7)^0, Z(7), Z(7)^2, Z(7)^3, Z(7)^4, Z(7)^5 ],
  [ 0*Z(7), Z(7)^2, Z(7)^3, Z(7)^4, Z(7)^5, Z(7)^0, Z(7) ],
  [ 0*Z(7), Z(7)^4, Z(7)^5, Z(7)^0, Z(7), Z(7)^2, Z(7)^3 ] ]
gap> oa:=DifferenceSetToOrthogonalArray(ds);
[ [ 1, 2, 3, 4, 5, 6, 7 ], [ 2, 4, 6, 3, 1, 7, 5 ],
  [ 3, 6, 5, 7, 4, 1, 2 ], [ 4, 3, 7, 6, 2, 5, 1 ],
  [ 5, 1, 4, 2, 7, 3, 6 ], [ 6, 7, 1, 5, 3, 2, 4 ],
  [ 7, 5, 2, 1, 6, 4, 3 ], [ 1, 4, 5, 6, 7, 2, 3 ],
  [ 2, 3, 1, 7, 5, 4, 6 ], [ 3, 7, 4, 1, 2, 6, 5 ],
  [ 4, 6, 2, 5, 1, 3, 7 ], [ 5, 2, 7, 3, 6, 1, 4 ],
  [ 6, 5, 3, 2, 4, 7, 1 ], [ 7, 1, 6, 4, 3, 5, 2 ],
  [ 1, 6, 7, 2, 3, 4, 5 ], [ 2, 7, 5, 4, 6, 3, 1 ],
  [ 3, 1, 2, 6, 5, 7, 4 ], [ 4, 5, 1, 3, 7, 6, 2 ],
  [ 5, 3, 6, 1, 4, 2, 7 ], [ 6, 2, 4, 7, 1, 5, 3 ],
  [ 7, 4, 3, 5, 2, 1, 6 ] ]
```

Projection cubes of symmetric designs

```
gap> c:=OrthogonalArrayToCube(oa);
[ [ [ [ [ [ [ 0, 0, 0, 0, 0, 0, 0 ], [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ] ],
            [ [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ] ],
            [ [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ],
                [ 0, 0, 0, 0, 0, 0, 0 ] ],
```

A $\mathcal{P}^3(16, 6, 2)$-cube with non-isomorphic projections



The End

Thanks for your attention!

The End


Paška sirana d.d. Pag
Paški sir
tvrdi sir od ovčjeg mlijeka
Sheep Milk Hard Cheese
Paška sirana Pag
Pravi bokun sira