Exercise 2

- Construct a permutation group with an action on tuples
- Consider the group $GL(4, 53)$ as a permutation group and compute its order.
- Consider the following group:

  $$G := \langle a, b, c, d, e, f \mid a^2, d^3, e^3, b*e*b^{-1}*e, b^4, e^{-1}*a*e*a, b^{-1}*d*b*d, \\ b^{-1}*a*b*a, (a*f)^2, f^{-1}*b^{-1}*f*b, d^{-1}*a*d*a, \\ f^{-1}*d^{-1}*f*d, f^{-1}*e^{-1}*f*e, f^5, (e^{-1}*d)^3, (d^{-1}*e^{-1})^3 \rangle$$

  This group is also available in the github repository under `Exercises2/mysterygroup.g`: `https://github.com/MeikeWeiss/GAP-Days2025-Intro`
  Test the following properties:

  - The order of the group
  - If the group is nilpotent
  - If the group is solvable
  - The derived series and its factors
  - Is this group polycyclic?
  - Is the group elementary abelian?

  Can you completely characterise the group?

- More exercises from last year can be found here: https://www.ilariacolazzo.info/gap/tutorials/sheet2/.