

Exercise 2

- Construct a permutation group with an action on tuples
- Consider the group $GL(4, 53)$ as a permutation group and compute its order.
- Consider the following group:

$$G := \langle a, b, c, d, e, f \mid a^2, d^3, e^3, b * e * b^{-1} * e, b^4, e^{-1} * a * e * a, b^{-1} * d * b * d, \\ b^{-1} * a * b * a, (a * f)^2, f^{-1} * b^{-1} * f * b, d^{-1} * a * d * a, \\ f^{-1} * d^{-1} * f * d, f^{-1} * e^{-1} * f * e, f^5, (e^{-1} * d)^3, (d^{-1} * e^{-1})^3 \rangle$$

This group is also available in the github repository under `Exercises2/mysterygroup.g`:
<https://github.com/MeikeWeiss/GAP-Days2025-Intro>

Test the following properties:

- The order of the group
- If the group is nilpotent
- If the group is solvable
- The derived series and its factors
- Is this group polycyclic?
- Is the group elementary abelian?

Can you completely characterise the group?

- More exercises from last year can be found here:
<https://www.ilariacolazzo.info/gap/tutorials/sheet2/>.