Exercise-Sheet A

Polycyclic groups

April 2025

Exercise 1: Let U be a Sylow 5-subgroup in $Sym(99)$, let $N = N_{Sym(99)}(U)$ and let $F = N/U$.

(a) Determine a Pc-presentation for F.
(b) Is F nilpotent or abelian? What is the derived length of F?
(c) How many conjugacy classes of non-normal maximal subgroups does F have?

Exercise 2: For $n \in \mathbb{N}$ let $f(n)$ denote order of a smallest non-abelian group with exactly n conjugacy classes. Determine $f(n)$ for $3 \leq n \leq 14$.

Exercise 3: Let F be free on a, b and let $R = [[a, b], b]]$. Show that the class-10 quotient of F/R is torsion free.

Exercise 4: For $1 \leq n \leq 30$ determine the number of solvable transitive groups of degree n up to conjugacy. How many of the groups are nilpotent?

Exercise 5: Determine the smallest solvable group of derived length 4. Can you find a small group of derived length 5?