New features of numericalsgps

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The package numericalsgps

Authors

This package was started by M. Delgado, PAGS and J. J. Morais.
The maintainers are now M. Delgado and PAGS.

Aim

Provide computational tools to work with numerical semigroups, that is, submonoids of $(\mathbb{N}, +)$ with finite complement in $\mathbb{N}$.

Contents of the package

Main

- Definitions and notable elements
- Basic operations
- Presentations
- Constructing numerical semigroups from others
- Irreducible numerical semigroups
- Ideals
- Maximal embedding dimension
- Nonunique invariants for factorizations

Apendices

- Generalities
- Random functions
- Contributions
  A. Sammartano (Purdue), C. O'Neill (Duke)
- References

New

- Polynomials and numerical semigroups

Bugs corrected/improvements for the new release

- Corrected InfoLevel 0 (now warning)
- Changed connectedcomponents in minimal presentations with isconnected (in presentaciones.gi); now using adjacency matrix
- Changed BettiElements accordingly (now there is no need of $\mathcal{R}$-classes)
- ReducedSetGeneratorsOfNumericalSemigroup moves to a synonym
- Fixed error in MultiplicityOfNumericalSemigroup for some proportionally modular numerical semigroup
- Fixed small (probable) bug in the output of AsGluingOfNumericalSemigroups
- FactorizationsInteger returned (it uses RestrictedPartitions)

  ```
  gap> FactorizationsIntegerWRTList(2,[1,1]);
  [ [ 2, 2 ], [ 2, 2 ], [ 2, 2 ] ]
  ```

  now the output is correct for lists with repeated elements
- Improved OmegaPrimalityOfElementInNumericalSemigroup (Chris O'Neill)
- Chapters in manual related to ideals and minimal presentations reorganized

Functions implemented for the new release (factorizations)

- DenumerantElementInNumericalSemigroup
- IsSuperSymmetricNumericalSemigroup
- IsAdditiveNumericalSemigroup
- AdjustmentOfNumericalSemigroup
- MaximalDenumerantOfNumericalSemigroup
- MaximalDenumerantOfSetOfFactorizations
- MaximalDenumerantOfElementInNumericalSemigroup
- FactorizationsElementListWRTNumericalSemigroup (O'Neill)
- OmegaPrimalityOfElementListInNumericalSemigroup (O'Neill)
- MoebiusFunctionAssociatedToNumericalSemigroup
- HomogeneousCatenaryDegreeOfNumericalSemigroup
- HomogeneousBettiElementsOfNumericalSemigroup (would benefit from Singular)
- FactorizationsInHomogenizationOfNumericalSemigroup
- BelongsToHomogenizationOfNumericalSemigroup

Functions implemented for the new release (polynomials)

- NumericalSemigroupPolynomial
- HilbertSeriesOfNumericalSemigroup (added on the plane)
- GraeffePolynomial
- IsCyclotomicPolynomial
- IsKroneckerPolynomial
- IsCyclotomicNumericalSemigroup
- IsSelfReciprocalUnivariatePolynomial
- SemigroupOfValuesOfPlaneCurveWithSinglePlaceAtInfinity
- IsDeltaSequence
- DeltaSequencesWithFrobeniusNumber
- CurveAssociatedToDeltaSequence

Functions implemented for ideals

- StarClosureOfIdealOfNumericalSemigroup

Functions implemented but not included (with A. Sánchez-R.-Navarro)

- AdjacentCatenaryDegreeOfSetOfFactorizations (added on the plane)
- EqualCatenaryDegreeOfSetOfFactorizations (added on the plane)
- MonotoneCatenaryDegreeOfSetOfFactorizations (added on the plane)
- MonotonePrimitiveElementsOfNumericalSemigroup
- PrimitiveElementsOfNumericalSemigroup
- EqualPrimitiveElementsOfNumericalSemigroup
- AdjacentCatenaryDegreeOfNumericalSemigroup
- EqualCatenaryDegreeOfNumericalSemigroup
- MonotoneCatenaryDegreeOfNumericalSemigroup

Why not included so far?

Most of them need to find a Graver basis for a system of linear Diophantine equations, or the set of nonnegative integer solutions to a system of linear equations.

- We have tried our own algorithms: work nice in a few cases, for instance $O(log)$ for three three variables; but too slow for most of the practical settings
- We have implemented Contejean and Devie algorithm... again too slow
- We have tried `4ti2Interface` by S. Gutsche: works fine, but our (potential) users will get in to trouble to install 4ti2 and get it running
- We would like `numericalsgps` to be standalone, or at least to be able to test if some functions are not worth to be used if the user does not have the appropriate tools installed
- So we need HELP

Affine semigrops

- Computing the set of factorizations is just solving over the nonnegative integers as system of the form $Ax = b$ (with nonnegative integer coefficients)
- So a tool for solving this would allow us to extend many of the functions already implemented for numerical semigroups to the more general setting of affine semigroups
- Also singular can be used (by eliminating variables) to compute a presentation of any affine semigroup

THANK YOU!