NormalizInterface

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Aachen, 25.08.2014

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Normaliz

Developed by W. Bruns, B. Ichim, T. Römer, C. Söger.

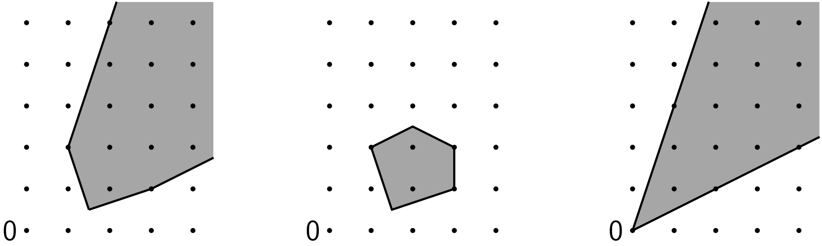
- Open source software (GPL)
- written in C++ (using Boost and GMP/MPIR)
- parallelized with OpenMP
- runs under Linux, MacOs and MS Windows
- C++ library libnormaliz
- file based interfaces for Singular, Macaulay 2 and Sage
- C++ level interfaces to CoCoA, polymake, Regina and GAP
- GUI interface jNormaliz

Normaliz has found applications in commutative algebra, toric geometry, combinatorics, integer programming, invariant theory, elimination theory, mathematical logic, algebraic topology and even theoretical physics.

Rational Polyhedra

Definition

A (rational) polyhedron P is the intersection of finitely many (rational) halfspaces. If it is bounded, then it is called a polytope. If all the halfspaces are linear, then P is a cone.



Input to Normaliz by

- generators: vertices and/or rays, or
- constraints: homogeneous or inhomogeneous equations, inequalities, congruences.

The tasks of Normaliz: Hilbert basis

Assume C is a pointed cone.

Theorem (Gordan's Lemma)

Let $C \subset \mathbb{R}^d$ be a rational cone. Then $C \cap \mathbb{Z}^d$ is an affine monoid, i.e. a finitely generated submonoid of $\mathbb{Z}^d$.

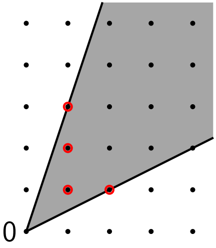
Normaliz computes the unique minimal finite system of generators of $M = C \cap \mathbb{Z}^d$, the Hilbert basis

$$\mathrm{Hilb}(M).$$

Normaliz has two algorithms for Hilbert bases:

- the original Normaliz algorithm,
- a variant of an algorithm due to Pottier (dual algorithm).

($\mathbb{Z}^d$ can be replaced by a sublattice L.)



The tasks of Normaliz: Hilbert series

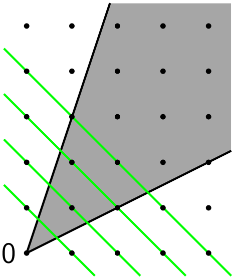
A grading on M is a surjective $\mathbb{Z}$-linear form $\deg : \mathrm{gp}(M) \to \mathbb{Z}$ such that $\deg(x) > 0$ for $x \in M$, $x \neq 0$

The Hilbert (or Ehrhart) function is given by

$$H(M,k) = \#\{x \in M : \deg x = k\}$$

and the Hilbert (Ehrhart) series is

$$H_M(t) = \sum_{k=0}^{\infty} H(M,k)t^k.$$



Theorem (Hilbert-Serre, Ehrhart)

- *$H_M(t)$ is a rational function*
- *$H(M,k)$ is a quasi-polynomial for $k \geq 0$*

NormalizInterface for GAP

In development with Sebastian Gutsche and Max Horn.

- (almost) full access to libnormaliz
- the GAP object NmzCone encapsulates a libnormaliz cone
- first interactive interface to libnormaliz
- still work in progress

DEMO